of the intellect break down. They either beg the question at once, in that intellect is defined in such a way as to mean something which cannot give truth, or they involve an appeal to the intellect the incompetence of which they are designed to demonstrate. On the other hand, Bergson's intuition, which is to give relief from the deadlocks which the intellect creates, will do nothing of the sort. Not only does it raise more difficulties than it solves, but it can only be expressed by reference to the intellect and the objects of the intellect.

The reason for this collapse of the Bergsonian philosophy is obvious. Bergson has only repeated the mistakes for which he reproaches Kant. In order to save the freedom of the will, God and immortality from all possible assaults of the intellect, Kant put these realities outside all possible knowledge. In much the same way, in order to have an answer to all possible difficulties which the intellect creates (and because he is apparently ignorant of the intellectual solution of certain classic difficulties), and in order to be able to say that what gives truth is not intellect, Bergson has to make intellect and intuition radically opposed to one another. But having so separated intellect and intuition, Bergson cannot justify either of them. He has not dealt fairly with intellect and has restricted it beforehand to that which is assumed not to be real. At the same time every attempt which Bergson makes to apply his doctrine of intuition, to show why intuition is necessary to supplement intellect, contradicts his own account of the nature of intuition.

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ANYNESS AND PURE FORM.

On another page in this issue, Prof. E. H. Strange criticizes Bergson's theory of intuition and derives it from Bergson's opposition to Kant's idealism.

In this connection it is appropriate to state that Kant bases his philosophy on the consideration that the highest laws of nature are identical with the mathematical or purely formal theorems. The latter are verified and indeed created by pure reflection, which means they are mind-made; or, as Kant expresses himself, they are products of *a priori* thought, they are transcendental, they serve us as the forms with whose help we reduce sense-impressions to well regulated experiences.

Kant asks in his Prolegomena,[1] "How is the agreement between the highest laws of nature and the theorems of purely formal thought possible?" and he sees only two possibilities. Either, says Kant, we find these laws in nature by experience or the mind makes these laws, and his answer is that the human intellect is so constituted that it can see the world only as its own tools shape it. Kant declares that the world of material things surrounding us can be recognized by the mind only according to the mind's constitution, not as the things are in themselves. The mind imposes its own laws upon the objective world. The opposite view, that the mind has derived its laws from the objective world, is excluded because we know positively that mathematics are mind-made, they are *a priori*. We can construct all mathematics without appealing to any experience of the senses.

Crusius, a German contemporary of Kant, proposed the theory that some world intellect, the creator or God, has established a preconceived harmony between mind and the universe, equipping the mind of man with such a mentality as to enable him to build up the highest (the purely formal) laws of the world constitution out of his own mental resources—a proposition which is quite plausible before a tribunal of theologians, but scarcely acceptable to philosophers.

Now comes Bergson, and having gone through a study of Kant (according to Professor Strange) he finds himself nonplused by Kantian idealism, and he sees another, a fourth, way out of the dilemma. On the basis of a misconceived interpretation of evolution he proposes that "intellect and matter have progressively adapted themselves to one another to attain at last a common form" (*Creative Evolution,* p. 217). This fourth possibility as proposed by Bergson is probably the most unfortunate theory of all, for it presupposes the notion that neither the highest laws of nature nor the truths of mathematical propositions have been stable.

Bergson seems to assume that the highest laws of nature as well as mathematical theorems were loose rules in the beginning and have gradually hardened into definite norms. The intellect and the material world have been in contact and have influenced each other. Our observation of the stars has gradually impressed itself upon their movements so as to assume more and more a definite mathematical form. Finally Kepler succeeded in sum-

[1] This and the following quotations are from memory.

marizing their motions in definite mathematical formulas. Before the mind was in touch with them they may have had other uniformities, or lack of uniformity, of motion. On the other hand the mind was rambling at first and mathematical theorems varied; but gradually they assumed definite form, and now a thinking being can evolve them out of the resources of his own mind by *a priori* argument.

This kind of interpretation of the agreement between mind and nature by a mutual adaptation of the intellect on the one side and the objective world on the other, displays a lack of insight into the very nature of mathematics, and misconceives also the character of natural law.

Take for instance the simple *a priori* statement that $2 \times 2 = 4$. Can there ever have been a time in which this statement was not true? There was a time indeed when the mind could not think in figures at all, when an arithmetical equation or an algebraic formula or a geometrical theorem must have been unmeaning to a sentient being. Indeed formal thoughts are still void of meaning to animals and are above the comprehension of savages; nevertheless their truth is established, and the celestial bodies moved according to the laws of Kepler before mankind originated and mathematical theorems were ever constructed. Kepler discovered his three laws; he did not invent them. To think that the objective truth of the highest laws of nature originated through a process of evolution indicates a misunderstanding not only of the very nature of mathematics, but also of the theory of evolution, and finally also of science itself.

We believe that Kant raised the problem of problems in philosophy, and explained his reasoning in his *Prolegomena,* which therefore, in our opinion, is the most important book that came from his pen. Kant's significance and the prominent place he holds in philosophy are due to the fact that he put his finger on the critical question, though he did not succeed in answering it. He established beyond the shadow of a doubt the apriority of all the formal sciences, but he explained this truth wrongly and has thus given rise to a wrong idealism, deriving therefrom an agnosticism which he formulated in the doctrine that things in themselves are unknowable. His disciples have come to the conclusion that things in themselves do not exist, and we suggest that what he really meant were "forms in themselves" viz., the Platonic ideas or types of things and they are not unknowable.

In consideration of the significance of the Kantian problem, which was suggested to him by Hume's skepticism as to the universal validity of the law of causation, we have published a translation of Kant's *Prolegomena* with our own criticism, substituting for Kant's solution our own which is the basis of the philosophy of science.

We grant that all formal knowledge, including logic and mathematics, is *a priori*, but the conditions for a construction of mathematics after all presuppose experience and the basis of mathematics is the creation of an abstract realm of pure form. For a construction of the purely formal sciences we exclude everything particular and concrete, matter as well as energy, and retain only our own activity with a scope of pure motion which involves the possibility of constructing pure interrelations. In other words, the tools with which we operate are ultimately derived from experience. We retain our ability to operate, our activity, our mode of moving about, but we move in a field void of particularity, a field which therefore can be applied anywhere.

We insist that mathematics and all the other purely formal sciences are not constructed from nothingness; they are ultimately based on experience. But from this experience is excluded everything that pertains to sense-experience, and we produce in this way a domain in which we construct relations that do not contain particulars, but outline conditions which apply anywhere to any place and to any time, and we have called this field of pure thought "anyness." The very term anyness contains an explanation of why these propositions can be applied anywhere, and this application anywhere involves that *a priori* propositions are both (as Kant rightly declares) universal and necessary.

By understanding the full significance of anyness, we understand also that these laws of pure form must apply to any possible world, real or imaginary. Thus we can in pure thought deduce the inevitable results of conditions under any circumstances, and we can understand that if there is a world of concrete materiality, its motions, constructions, formations and results of any kind of actions are—so far as their forms, their relations, are concerned—predetermined by the laws of pure form, viz., by the laws of anyness.

Thus harmony must obtain between the purely formal laws as we have produced them by *a priori* construction and as they appear in the concrete world of reality, because the two are the same. Suppose two mathematicians construct a parabola with the same co-

ordinates, would they not both come to the same conclusions as to the form of the parabola? And suppose that in reality a comet is determined by forces which possess a one-to-one correspondence with these same coordinates, would not the path of the comet possess a one-to-one correspondence to the figures of the parabolas of the two astronomers? The determinedness of all purely formal constructions is truly universal and applies anywhere in the domain of mathematics or pure thought and in any possible real world, also in this our world, i. e., the universe in which we live.

Considering the immanent necessity of the laws of form we can understand that this pre-established harmony has not been made by some supernatural being nor can it have originated gradually by a process of evolution, but it is intrinsically necessary. It is the immanent order which is the condition both of our natural laws and the intelligibility of existence. It is that same intrinsic regularity which can be observed everywhere in nature. This same regularity in the domain of form makes it possible that rational beings originate, that science can be established, that ideals can be proposed and lived up to, that a code of morality and a norm of right conduct can be formulated, and that the universe presents itself as a well-regulated and law-ordained cosmos.

A revision of almost all problems of philosophy from our standpoint will shed new light on their solutions, as will appear when we consider Prof. Hartley B. Alexander's article on "The Definition of Number." When enumerating the different conceptions of the interrelation between logical and mathematical views on the one side and philosophy on the other, he omits to mention the solution offered by the philosophy of form, which alone can be regarded as the philosophy of science.

Mr. Bertrand Russell sees the most essential feature of mathematics in its logical interrelations and goes so far as to claim that mathematics has nothing to do with space. Without objecting to definitions we prefer to regard at least geometry as the purely formal science of extension, which means space, not real space but pure or mathematical space. Mathematics presupposes logic and contains one additional element which is commonly called space, but like all purely formal sciences mathematics produces its objects of investigation by *a priori* construction. The elements with which we start are products of abstract thought in the realm of pure form. created by thinking away everything that is particular, viz., all concrete objects that consist of matter and energy. Thus we retain

the idea of pure motion and a possibility of establishing pure interrelations.

Pure motion means a change of place without implying energy, and a possibility of pure interrelations is a field of pure motion. We start with these two abstract notions, on the part of the subject an ability to move about, on the part of the object, (i. e., the surrounding world), emptiness; and this emptiness offers a field of possible motion. With these conditions we construct whatever we may be pleased to build up, and observe the result.

In geometry we do something and note what will come of it. For instance, we move and note the trace of our motion. We call it a line. We move again and again, and let the traces of other lines enclose a space; we call the result a figure. Where two lines cross we have a point.

The system under construction may be Euclidean or non-Euclidean according to our start, whether or not we assume we are able to draw straight lines in the Euclidean space.[2] If in our plan of construction we exclude the straight line, we will have to move according to a definite principle in curves of a predetermined constant deviation, in which case our system will be different from the system of Euclid.

If two straight lines cross, the product of our construction is an angle, or rather four angles. The peculiarity of mathematics is to watch and observe the inevitable results of our own constructions, but the main characteristic of our constructions is this, that they are made in a field of anyness, i. e., they apply to any kind of construction made in the same way, not only in emptiness, but in any kind of a world filled with any kind of matter or any kind of energy.

The nature of matter and energy can only be discovered by experience through the senses, but the nature of pure interrelations can be determined by building up constructions in a field of anyness, as they must be under any conditions, which means under all conditions. Therefore the laws of pure form (in other words, the laws of anyness) will be valid for any kind of a world.

Thus we have an explanation why the theorems of pure mathematics are hyperphysical truths, and here we have a specimen of the nature of what theology has called the supernatural. There is only this difference between the old conception of the supernatural and this new conception of it which for the sake of distinction

[2] For an *a priori* construction of the plane, the straight line and the right angle see the author's *Foundations of Mathematics*.

we call the "hyperphysical," that the latter is as clear and self-evident as the former is mysterious, hazy, bewildering and mystifying.

The consequence of this conception of mathematics need not be traced here in all details, but we feel assured that in the long run it will solve all the modern problems of philosophy and dispose of the troubles which have been caused by pragmatism, Bergsonianism, by the advocates of the principle of relativity, and also by the logisticians. EDITOR.

LOUIS COUTURAT (1868-1914).

Besides the carnage in battleships and trenches, the great European war carries with it many accidental by-products of disaster not to be overlooked when casting up the grand total of losses the world is suffering. In the early days of last August when the first commotion in the commercial arteries to and from Paris was at its height, a heavy automobile at full speed chanced to run down the carriage in which Louis Couturat was traveling, and his immediate death was the result. Though only forty-six years old he held first rank in France among scientific workers in the philosophy of language, the philosophy of mathematics, and especially in the more modern aspect of logic—for which he agrees with English logicians in preferring the term "logistic," now that this word is but little known in its earlier significations listed in the dictionaries.

M. Couturat was singularly well informed on many questions, but the particular power and quality of his mind lay in a gift for deductive reasoning combined with the most punctilious intellectual honesty that would never countenance a compromise with the truths of reason. All his work is especially remarkable for the clearness of its representation. His style is never sullied by glittering and bizarre phrases intended to attract attention and admiration, but which often seem to cover a multitude of sins in the way of vague ideas and loose reasoning.

Couturat was first known by his painstaking and illuminating exposition of the mathematical infinite (*L'infini mathématique,* 1896) in which he discusses the idea of number and analyzes the concepts of continuity and the infinite, refuting practically all of Renouvier's arguments against the latter. His research in this line familiarized him with all the writings of Leibniz, and his next published work was an edition of more than two hundred fragments